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Single Shooting and ESDIRK Methods for adjoint-based optimization of an oil reservoir

Andrea Capolei, Carsten Völcker, Jan Frydendall, John Bagterp Jørgensen

Department of Informatics and Mathematical Modeling
Technical University of Denmark

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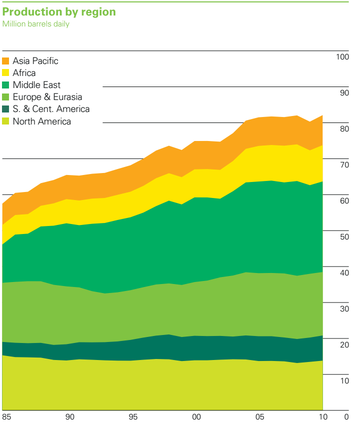
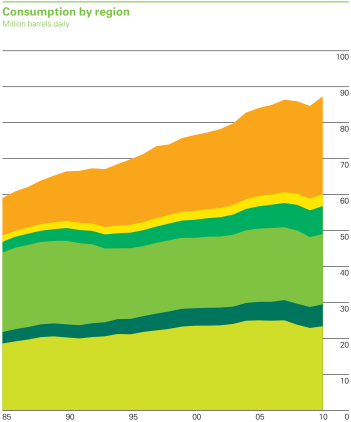
Outline

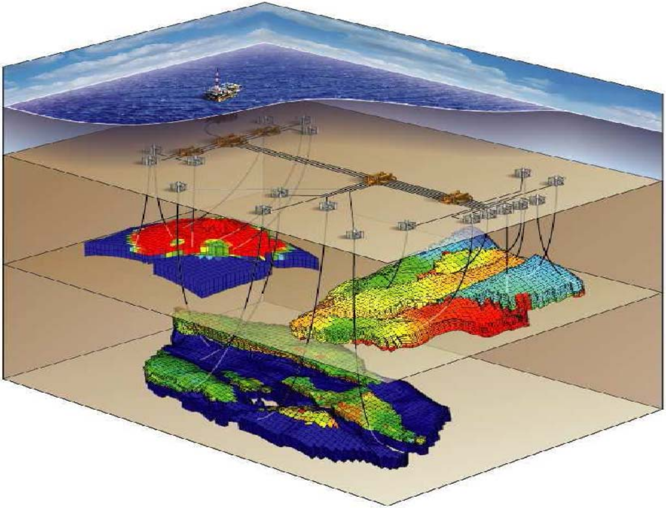
Increasing Oil demand



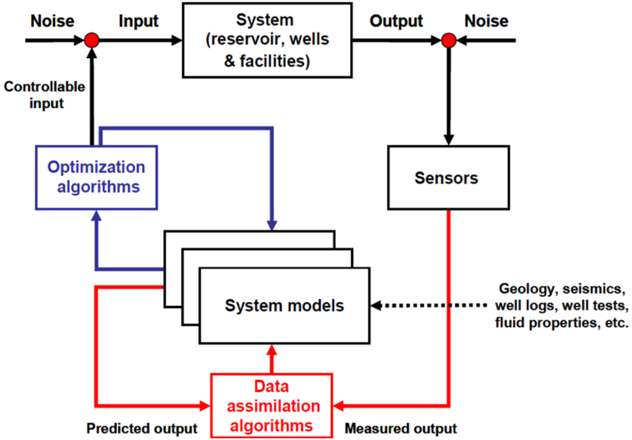


(from BP statistical review 2011)

An Offshore Oil Reservoir



Closed-loop reservoir management



(after Jansen 2005)

Water Flooding Modeled by a Two-Phase Flow Model

The mass conservation of water ($i \equiv w$) and oil ($i \equiv o$)

$$\frac{\partial}{\partial t} C_i(P_i, S_i) = -\nabla \cdot \mathbf{F}_i(P_i, S_i) + Q_i$$

The mass concentrations

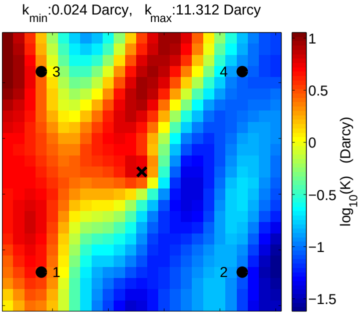
$$C_i = \phi \rho_i(P_i) S_i$$

Fluxes through the porous medium

$$\mathbf{F}_i = \rho_i(P_i) \mathbf{u}_i(P_i, S_i)$$

Darcy's law

$$\mathbf{u}_i(P_i, S_i) = -\mathbf{K} \frac{k_{ri}(S_i)}{\mu_i} \left(\nabla P_i - \rho_i(P_i) g \nabla z \right)$$



A general formulation of the two-phase flow problem

$$\frac{d}{dt} g(x(t)) = f(x(t), u(t))$$

Continuous form

Consider the continuous-time constrained optimal control problem in the Bolza form

$$\min_{x(t),u(t)} J = \hat{\Phi}(x(t_b)) + \int_{t_a}^{t_b} \Phi(x(t), u(t))dt \tag{1a}$$

subject to

$$x(t_a) = x_0 \tag{1b}$$

$$\frac{d}{dt} g\big(x(t)\big) = f(x(t), u(t)),\ t \in [t_a, t_b], \tag{1c}$$

$$u(t) \in \mathcal{U}(t) \tag{1d}$$

$x(t) \in \mathbb{R}^{n_x}$ is the state vector and $u(t) \in \mathbb{R}^{n_u}$ is the control vector. The time interval $I = [t_a, t_b]$ as well as the initial state, x_0, are assumed to be fixed.

Path constraints

Path constraints

$$\eta(x(t), u(t)) \geq 0 \tag{2}$$

are included as soft constraints using the following smooth approximation

$$\chi_i(x(t), u(t)) = \frac{1}{2}\left(\sqrt{\eta_i(x(t), u(t))^2 + {\beta_i}^2} - \eta_i(x(t), u(t)\right) \tag{3}$$

to the exact penalty function $\max(0, -\eta_i(x(t)))$ for $i \in \{1, \ldots, n_\eta\}$.With this approximation of the path constraints, the resulting stage cost, $\Phi(x(t), u(t))$, used in (11) consist of the inherent stage cost, $\tilde{\Phi}(x(t), u(t))$, and terms penalizing violation of the path constraints (2)

$$\Phi(x, u) = \tilde{\Phi}(x, u) + \|\chi(x, u)\|_{1,Q_1} + \frac{1}{2}\|\chi(x, u)\|^2_{2,Q_2} \tag{4}$$

Single Shooting Discretization

We introduce the function

$$\psi(\{u_k\}_{k=0}^{N-1}, x_0) = \Big\{ J = \int_{t_a}^{t_b} \Phi(x(t), u(t))dt + \hat{\Phi}(x(t_b)) : \quad x(t_0) = x_0, \\ \frac{d}{dt} g(x(t)) = f(x(t), u(t)),\ t_a \leq t \leq t_b, \\ u(t) = u_k,\ t_k \leq t < t_{k+1},\ k \in \mathcal{N} = 0, .., N-1 \Big\} \tag{5}$$

such that (1) can be approximated with the finite dimensional constrained optimization problem

$$\min_{y := \{u_k\}_{k=0}^{N-1}} \quad \psi = \psi(y, x_0) \tag{6a}$$

$$s.t. \quad u_{\min} \leq u_k \leq u_{\max} \qquad k \in \mathcal{N} \tag{6b}$$

$$\Delta u_{\min} \leq \Delta u_k \leq \Delta u_{\max} \qquad k \in \mathcal{N} \tag{6c}$$

$$c_k(u_k) \geq 0 \qquad k \in \mathcal{N} \tag{6d}$$

Sequential Quadratic Programming

We solve the NLP (6) iteratively improving a given estimate y^i of the solution by

$$y^{i+1} = y^i + \alpha^i p^i \tag{7}$$

where α^i $(0 < \alpha \leq 1)$ is determined by a linesearch (LS) strategy based on Powell's exact l_1-merit function. The search direction p^i is given by solving the KKT solution (p^i, λ^i, μ^i) of a quadratic approximation to (6)

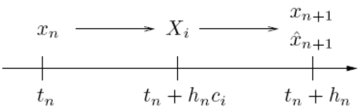
$$\begin{aligned} \min_p \ & \frac{1}{2} p' H^i p' + \nabla \psi(y^i)' p \\ s.t \quad & \nabla h(y^i)' p = -h(y^i) \\ & \nabla \bar{c}(y^i) \geq -\bar{c}(y^i) \end{aligned} \tag{8}$$

where $H^i \in \mathbb{R}^{n \times n}$ is an approximation for the Hessian $\nabla_y^2 L$ of the lagrangian function $L(y, \lambda, \mu) = \psi(y) - \lambda h(y) - \mu \bar{c}(y)$ which starting from an initial estimate H^0, is updated after every step using BFGS.

Runge-Kutta ESDIRK Methods

We use an embedded ESDIRK method

$$\begin{array}{c|cccccc}
0 & 0 & & & & \\
c_2 & a_{21} & \gamma & & & \\
c_3 & a_{31} & a_{32} & \gamma & & \\
\vdots & \vdots & & & \ddots & \\
1 & b_1 & b_2 & b_3 & \cdots & \gamma \\
\hline
x_{n+1} & b_1 & b_2 & b_3 & \cdots & \gamma \\
\hat{x}_{n+1} & \hat{b}_1 & \hat{b}_2 & \hat{b}_3 & \cdots & \hat{b}_s
\end{array}$$



1. Solve implicit system

$$T_i = t_n + h_n c_i, \quad i \in 2, \ldots, s$$

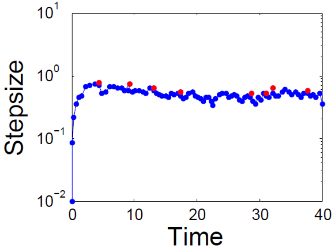
$$g(X_i) = g(x_n) + h_n \sum_{j=1}^{s} a_{ij} f(T_j, X_j, u)$$

2. Compute $x_{n+1} = X_s$
3. Compute the error/tolerance ratio

$$\begin{aligned} e_{n+1} &= g(x_{n+1}) - g(\hat{x}_{n+1}) \\ &= h_n \sum_{j=1}^{s} (b_j - \hat{b_j}) f(T_j, X_j, u) \end{aligned}$$

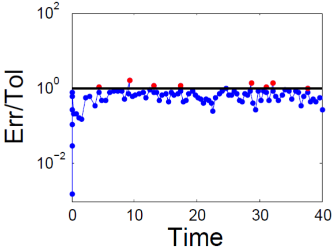
$$r_{n+1} = \frac{1}{\sqrt{n_x}} \left\| \frac{e_{n+1}}{atol + |g(x_{n+1})| rtol} \right\|_2$$

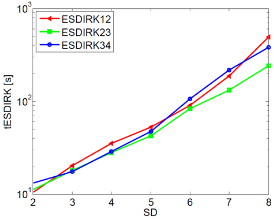
Temporal step size controller performance



Method : ESDIRK23
absTol : 1e−08
relTol : 0.0001

nStep : 96	nFun : 832
nFail : 9	nJac : 57
nDiv : 0	nLU : 76
nSlow : 1	nBack : 1662





(after Völcker 2010)

Proposition (Gradients based on Continuous Adjoints)

Consider the function $\psi = \psi(\{u_k\}_{k=0}^{N-1}; x_0)$ *defined by (5). The gradients,* $\partial\psi/\partial u_k$*, may be computed as*

$$\frac{\partial \psi}{\partial u_k} = \int_{t_k}^{t_{k+1}} \left(\frac{\partial \Phi}{\partial u} - \lambda^T \frac{\partial f}{\partial u} \right) dt \qquad k = 0, 1, \ldots, N-1 \tag{9}$$

in which $x(t)$ *is computed by solution of (1b)-(1c) and* $\lambda(t)$ *is computed by solution of the adjoint equations*

$$\frac{d\lambda^T}{dt} \frac{\partial g}{\partial x} + \lambda^T \frac{\partial f}{\partial x} - \frac{\partial \Phi}{\partial x} = 0 \tag{10a}$$

$$\frac{\partial \hat{\Phi}}{\partial x}(x(t_b)) + \lambda^T(t_b) \frac{\partial g}{\partial x}(x(t_b)) = 0 \tag{10b}$$

Remember that the dynamic model is given as

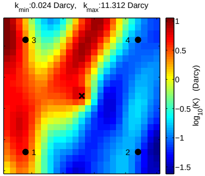
$$\frac{d}{dt} g(x(t)) = f(x(t), u(t))$$

Test case for waterflooding optimization

- squared reservoir of size $450m \times 450m \times 10m$
- uniform cartesian grid of $25x25x1$ grid blocks
- no flow boundaries, 4 injectors and 1 producer (rate controlled)
- We maximize an economic value for different discount factors $b \in 0, 0.06, 0.12$

$$\min_{x(t),u(t)} J(t_b) = -\text{NPV}(t_b) = \int_{t_a}^{t_b} \tilde{\Phi}(x(t), u(t))dt$$
$$\tilde{\Phi} = -\frac{1}{(1+b)^{t/365}} \sum_{j \in \mathcal{P}} \left(r_o(1 - f_{w,j}) - f_{w,j} r_w\right) q_j(t) \tag{11}$$

where $f_w = \lambda_w/(\lambda_w + \lambda_o), \quad \lambda_i = \rho_i k k_{ri}/\mu_i, \quad i \in \{w, o\}$



Constraints

The manipulated variable at time period $k \in \mathcal{N}$ is

$$u_k = \{\{q_{w,i,k}\}_{i \in \mathcal{I}}, \{q_{i,k}\}_{i \in \mathcal{P}}\} \tag{12}$$

with $\mathcal{I}$ being the set of injectors and $\mathcal{P}$ being the set of producers.

$q_{w,i,k}$	injection rate (m^3/day) of water at injector $i \in \mathcal{I}$
$q_{i,k}$	total flow rate (m^3/day) at producer $i \in \mathcal{P}$

Bound constraints

$$0 \leq q_{w,i,k} \leq q_{\max} \qquad i \in \mathcal{I},\ k \in \mathcal{N} \tag{13a}$$

$$0 \leq q_{i,k} \leq q_{\max} \qquad i \in \mathcal{P},\ k \in \mathcal{N} \tag{13b}$$

Rate constraints

$$|q_{i,k} - q_{i,k-1}| \leq 5 \qquad i \in \mathcal{I} \cup \mathcal{P},\ k \in \mathcal{N} \tag{14a}$$

$$|q_{w,i,k} - q_{w,i,k-1}| \leq 5 \qquad i \in \mathcal{I},\ k \in \mathcal{N} \tag{14b}$$

Constraints (2)

voidage replacement constraint

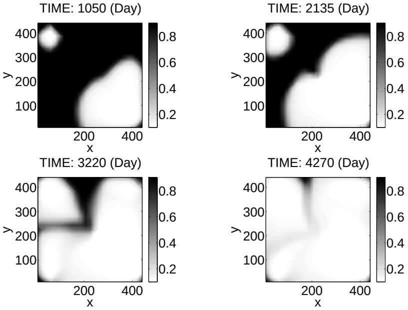
$$\sum_{i \in \mathcal{I}} q_{i,k} = \sum_{i \in \mathcal{I}} q_{w,i,k} = \sum_{i \in \mathcal{P}} q_{i,k} \qquad k \in \mathcal{N} \tag{15}$$

total injection constraint

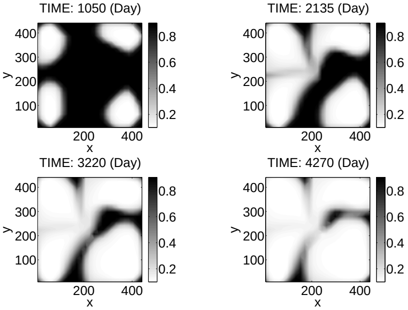
$$\sum_{i \in \mathcal{I}} q_{w,i,k} = q_{\max} \qquad k \in \mathcal{N} \tag{16}$$

- We set $q_{\max} = 100$ m^3/day, $t_b = 4270$ days, $T_s = 35$ days hence $N = 122$ periods.
- Injection of 1.05 pore volume during operation of the reservoir
- We consider as a reference case a fixed water injection of $100/4 = 25$ m^3/day from each injector.
- The prediction horizon t_b is optimal in the reference case

Optimal solution



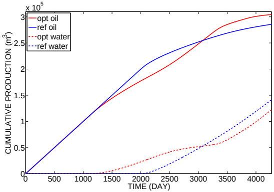
(a) Optimal solution $(b = 0)$.



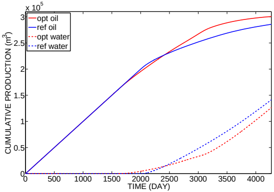
(b) Reference solution.

Oil saturations at different times for the optimal solution and the reference solution.

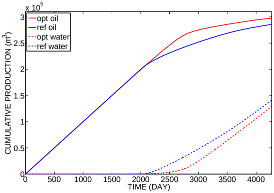
Cumulative oil and water production for different discount factors



(a) Discount factor $b = 0$.



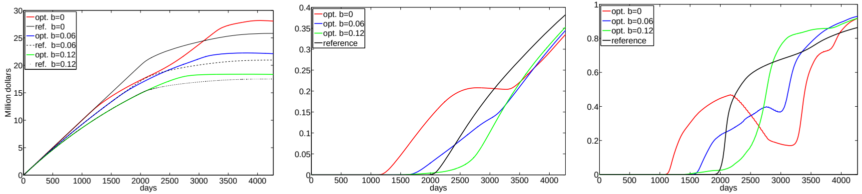
(b) Discount factor $b = 0.06$.



(c) Discount factor $b = 0.12$.

Cumulative oil and water productions for different discount factors, b.

NPV, Water cut, Water fraction



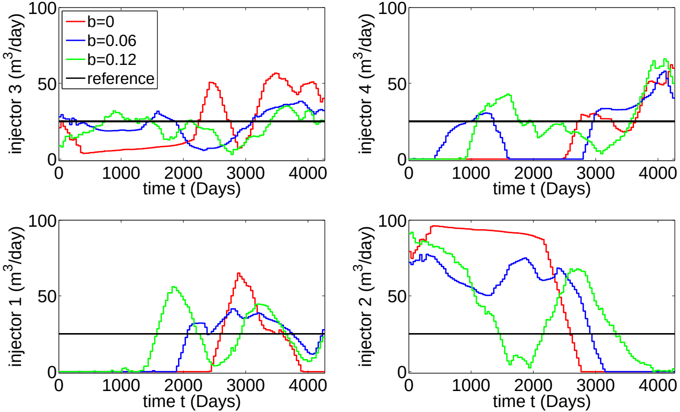
(a) NPV.

(b) Water cut.

(c) Water fraction (kg water/kg fluid).

The net present value (NPV), water cut (accumulated water production per produced fluid), and the water fraction as function of time for the scenarios considered.

Optimal input trajectories



Optimal water injection rates for different discount factors, b.

Improvement table

Table: Key indicators for the optimized cases. Improvements are compared to the base case.

b	NPV 10^6 USD	ΔNPV %	Cum. Oil 10^5 m^3	ΔOil %	Cum. water 10^5 m^3	ΔWater %
0	28.0	$+8.7$	3.05	$+6.5$	0.122	-13.2
0.06	22.1	$+5.6$	3.01	$+5.2$	0.126	-10.5
0.12	18.3	$+4.8$	2.98	$+4.1$	0.129	-8.2

b	Oil Rec. factor %	ΔOil Rec. factor %-point
0	83.7	$+5.2$
0.06	82.6	$+4.1$
0.12	81.7	$+3.2$

Conclusions

A novel algorithm for large scale optimal control based on

- a novel formulation of the differential equations
- an ESDIRK method for integration of differential equations
- the continuous adjoint method for gradient computation
- the SQP method for optimization
- the single shooting principle

This algorithm is applied for production optimization of oil reservoirs. For this field, optimal control improves the NPV by 8.7%.

Optimal control and nonlinear model predictive control can potentially have a very big impact in oil reservoir management.

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www.cere.dtu.dk

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